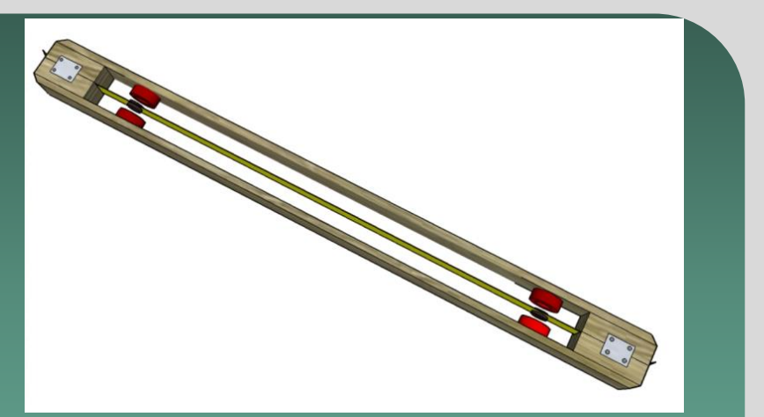
Limit Cycle Oscillations of a Pre-Tensed Membrane Strip



Msc Research Study by Ariel Drachinsky
Under the Guidance of Prof. Daniella Raveh

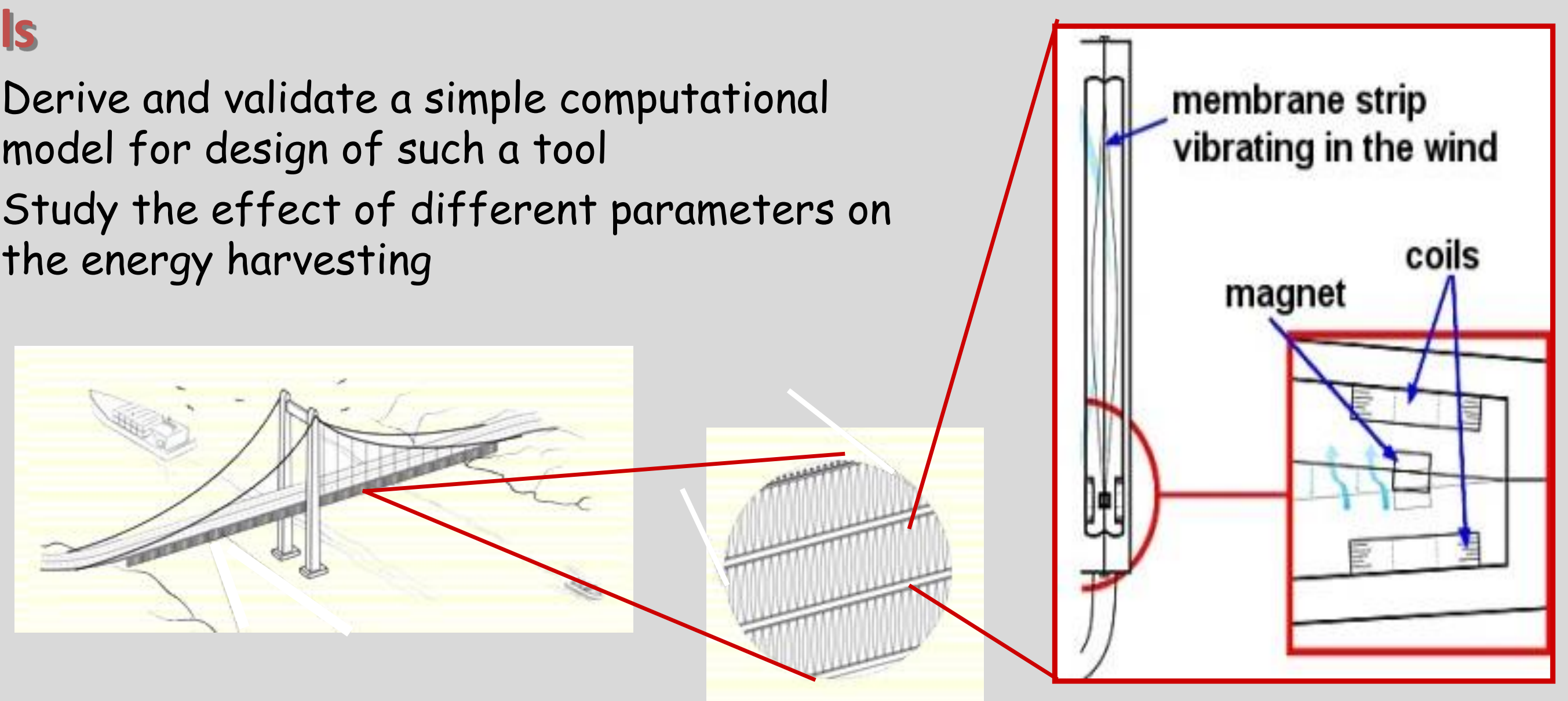
Introduction

Motivation

- Energy harvesting from flutter (LCO)
- Energy harvesting at low airspeed (<10m/s)
- Compact, cheap and green energy generator

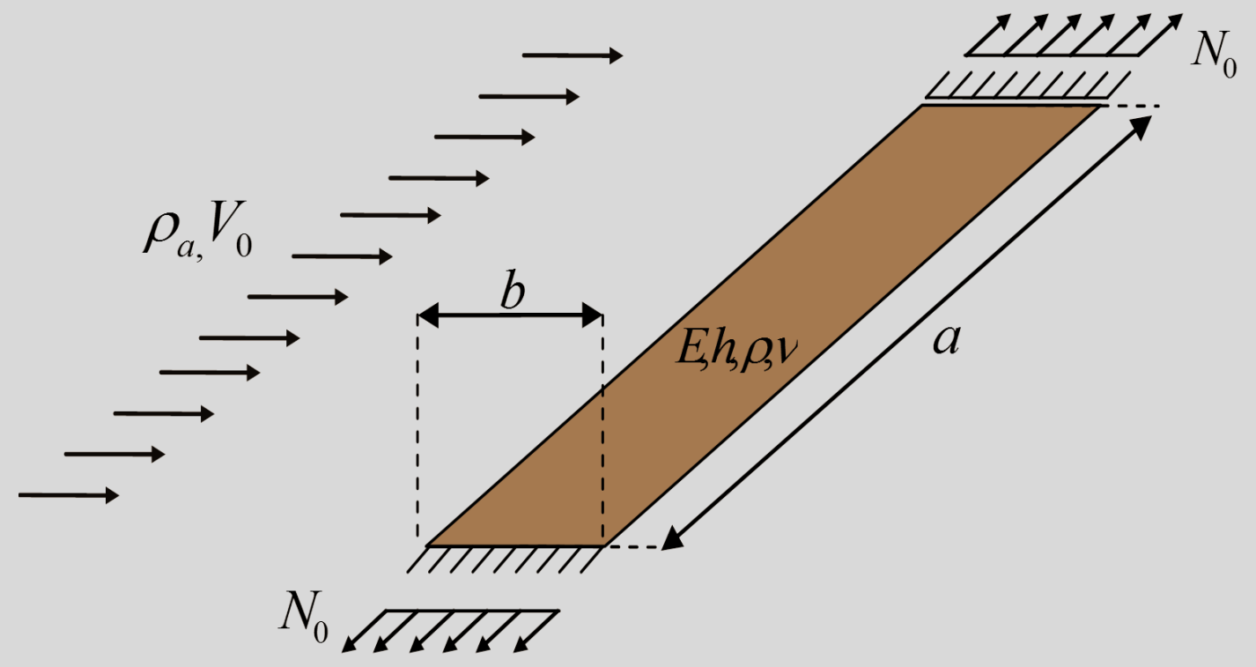
Goals

- Derive and validate a simple computational model for design of such a tool
- Study the effect of different parameters on the energy harvesting



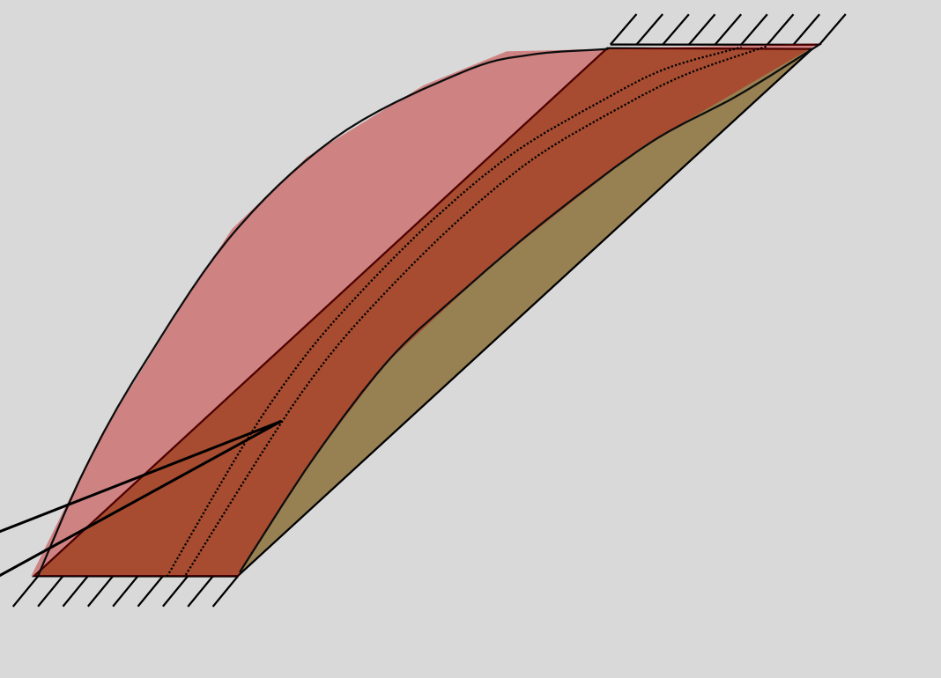
Problem Definition

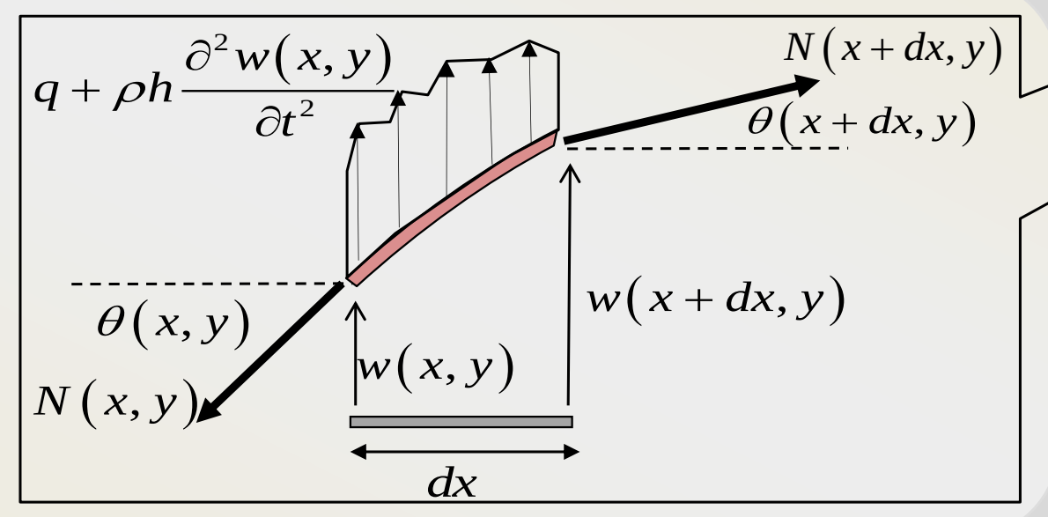
- A thin membrane of high AR is clamped at the short edges.
- The membrane is pretensed in the long (span) direction.
- Air flows in the direction of the short edge (chord).
- At a certain velocity, the membrane flutters.
- Due to non-linear stiffening effects the oscillations converge to a limit cycle.



Mathematical Mmodel “Beam-String Model”

- Linear, elastic material properties
- Chordwise bending is negligible
- Large deformations, small angles small strains
- Potential flow
- Oscillations reach steady state
- Energy harvesting system (coils) is not considered





Elastic term

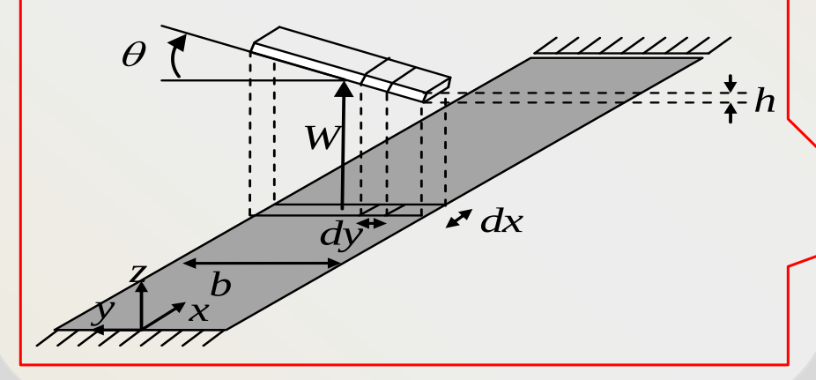
Bernouli-Euler bending

Saint-Venant’s torsion

$$\int_{-b/2}^{b/2} Q_e dy = EI\frac{\partial^4 W}{\partial x^4} \qquad \int_{-b/2}^{b/2} Q_e y dy = -GJ\frac{\partial^2 \theta}{\partial x^2}$$

No chordwise bending

$$w(x,y) = W(x) + \theta(x)\cdot y$$



Inertial term

$$\frac{\partial}{\partial x}\left(N(x,y)\frac{\partial w(x,y)}{\partial x}\right) = \rho h\frac{\partial^2 w(x,y)}{\partial t^2} + Q_a + Q_e$$

Nonlinear geometric stiffness term

Stiffness due to tenssion

- Pretenssion
- Additional tension due to large deformation

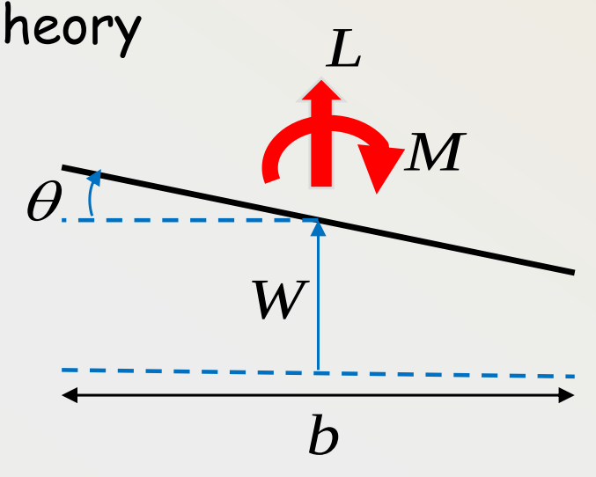
$$N(x,y) = N_0 + \Delta N = N_0 + Eh\frac{L-a}{a}$$

$$\approx N_0 + \frac{Eh}{2a}\int_0^a \left(\frac{\partial w(x,y)}{\partial x}\right)^2 dx$$

Aerodynamic term

theodorsen’s strip theory

$$\int_{-b/2}^{b/2} Q_a dy = L_a(ik), \qquad \int_{-b/2}^{b/2} Q_a y dy = M_a(ik)$$



$$\begin{Bmatrix} L \\ M \end{Bmatrix} = \left[K_a(\omega)\right]\begin{Bmatrix} W \\ \theta \end{Bmatrix} + \left[C_a(\omega)\right]\begin{Bmatrix} \dot W \\ \dot\theta \end{Bmatrix}$$

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Solution Via Galerkin’s Method

- Galerkins method transform s a PDE to a set of ODE’s

$$w = W + \theta\cdot y$$

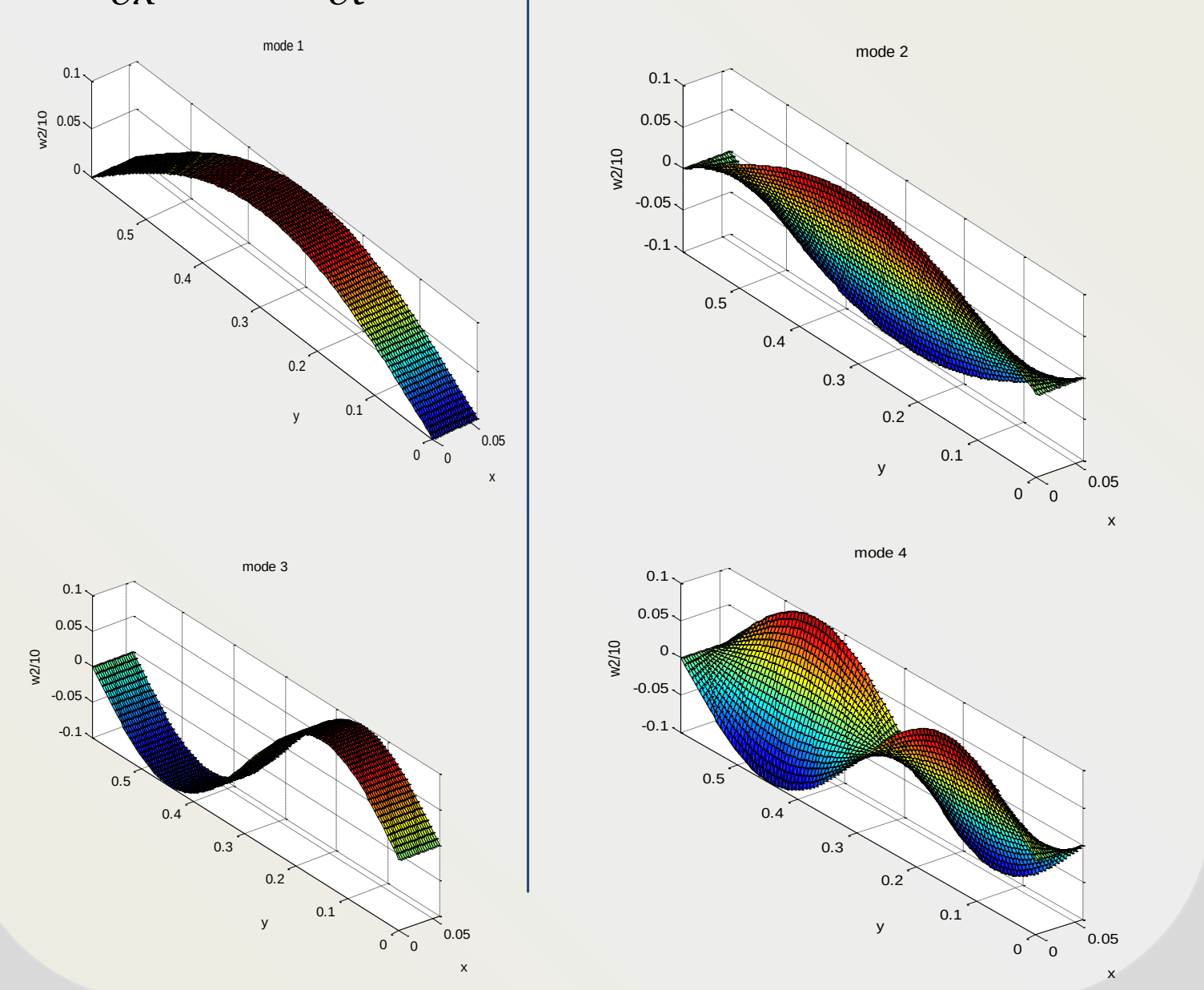
$$W(x,t) \approx X_1(t)W_1(x) + X_2(t)W_2(x) + X_3(t)W_3(x) + X_4(t)W_4(x)$$

$$\theta(x,t) \approx X_1(t)\theta_1(x) + X_2(t)\theta_2(x) + X_3(t)\theta_3(x) + X_4(t)\theta_4(x)$$

$$\int_0^a\int_{-b/2}^{b/2} \underbrace{\left[\frac{\partial}{\partial x}\left(N(x,y)\frac{\partial w(x,y)}{\partial x}\right) - \rho h\frac{\partial^2 w(x,y)}{\partial t^2} - Q_a - Q_e\right]}_{R} \cdot w_i dydx = 0$$

$$\{\ddot X\} + ([C] - v[C_a])\{\dot X\} + ([K] - v^2[K_a])\{X\} + \{f(\xi X^3)\} = 0$$

$$\sigma_0 A\frac{\partial^2 W}{\partial x^2} - \rho A\frac{\partial^2 W}{\partial t^2} = 0; \qquad (\sigma_0 I_p + GJ)\frac{\partial^2 \theta}{\partial x^2} - \rho I_p\frac{\partial^2 \theta}{\partial t^2} = 0$$

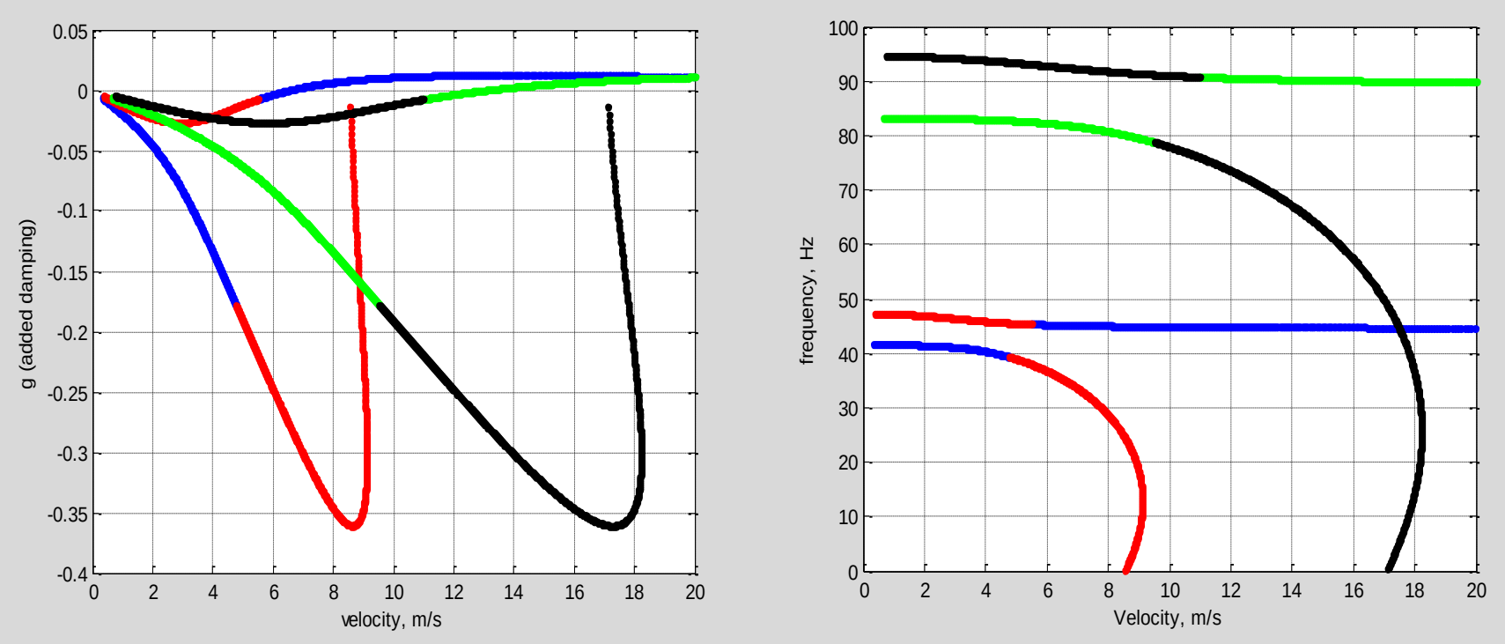


- Simulation time ~3000 periods. (small damping at near-flutter velocities)
- Time step ~0.05 period.
- Aerodynamic terms depend on frequency.->Solution is followed by a spectral analysis.
- Aerodynamic terms are updated, and the problem is simulated again until convergence.

Test-case

Property	a	b	h	σ_0	E	ρ	ζ	ρ_a	ν
value	596	25	0.25	3.89	0698	1430	0.0	1.225	0.39
units	mm	mm	mm	MPa	MPa	Kg/m³	%	Kg/m³	--

Linear analysis, comparison between 2d and 3d models

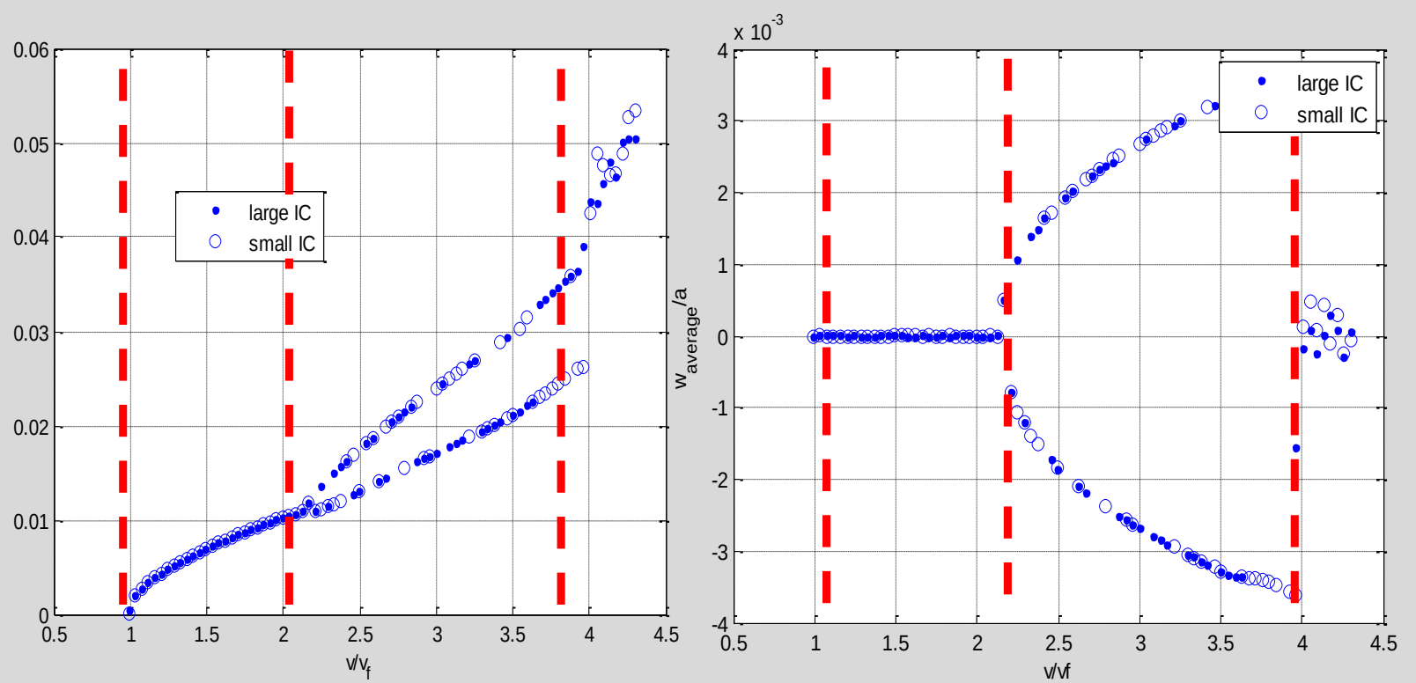


Vg plot obtained for the testcase

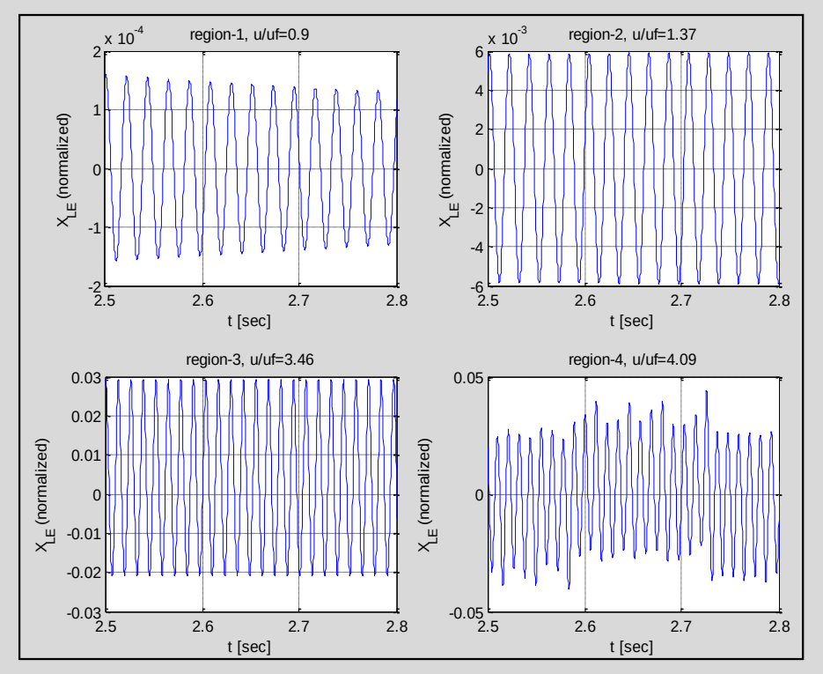
	four modes beam-string model	10 modes shell+panel model
1st Bending	43.9	44.2
1st Torsion [Hz]	49.1	48.7
2nd Bending [Hz]	87.9	88.4
2nd Torsion [Hz]	98.2	98.3
V_f [m/s]	6.2	6.4
V_d [m/s]	8.6	8.9

- ✓ Strip theory is adequate
- ✓ Beam model is adequate
- ✓ Bending stiffness is negligible
- ✓ Four modes are sufficient

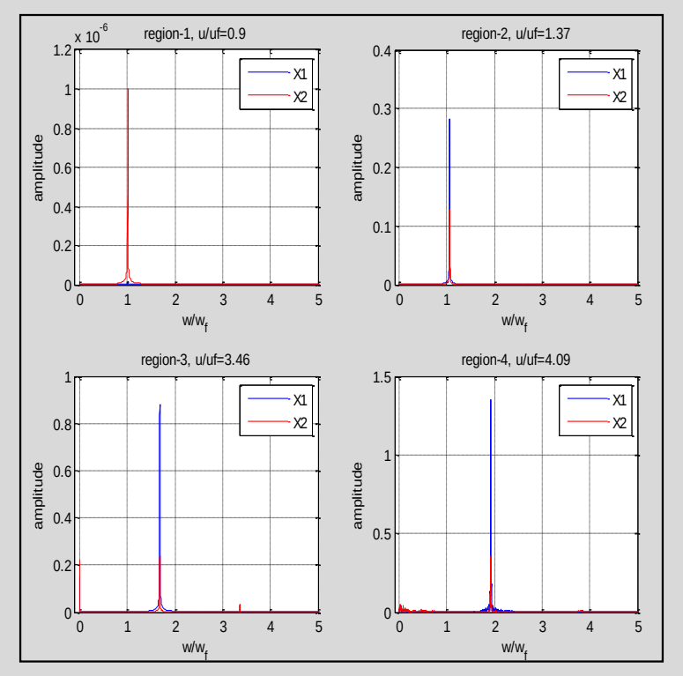
Non-linear analysis



Maximum and average displacement vs. airspeed



LE maximum displacement vs. time



Frequency content

region	v/vf	
1	0-1	Stable. All oscillations decay.
2	1-2.2	LCO 1st and 2nd modes.
3	2.2-3.7	LCO 1st and 2nd modes with a static offset (the oscillation is about non-zero deflection). Not obtained in experiment
4	>3.7	Gradual loss of periodicity, turning into chaotic oscillations.

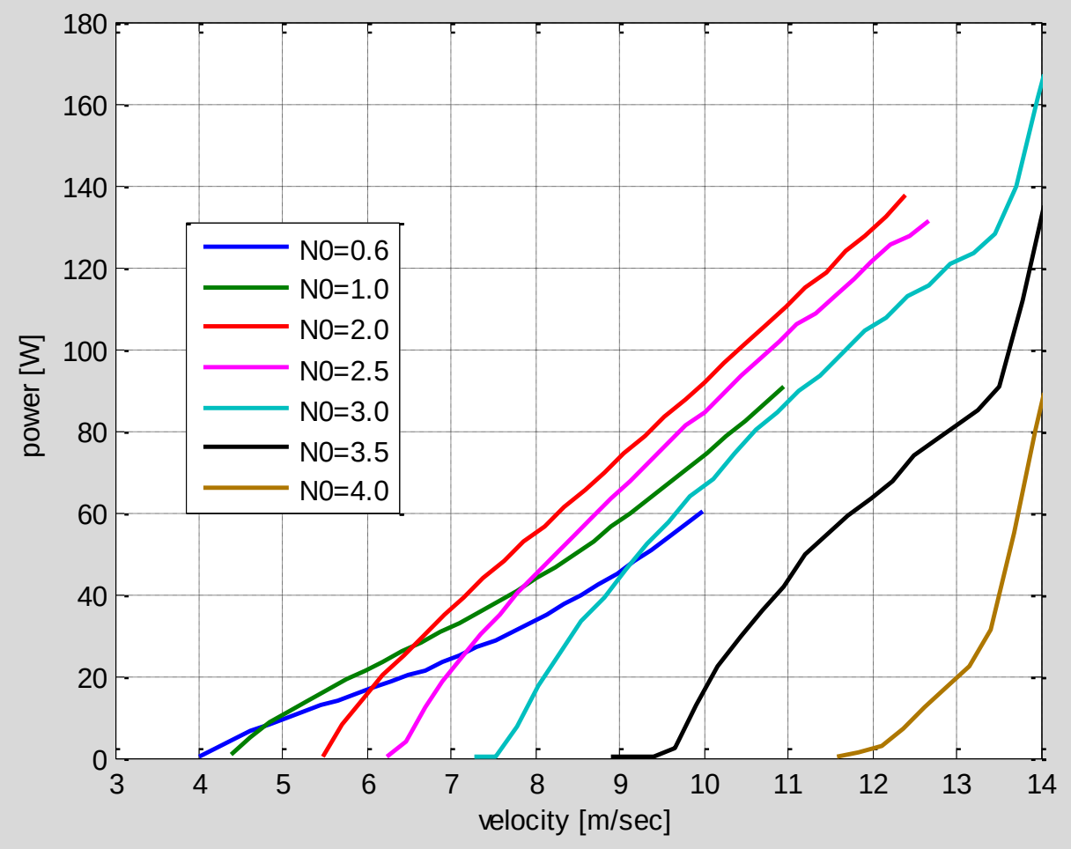
Parametric study, effect of preload on power

Power calculation

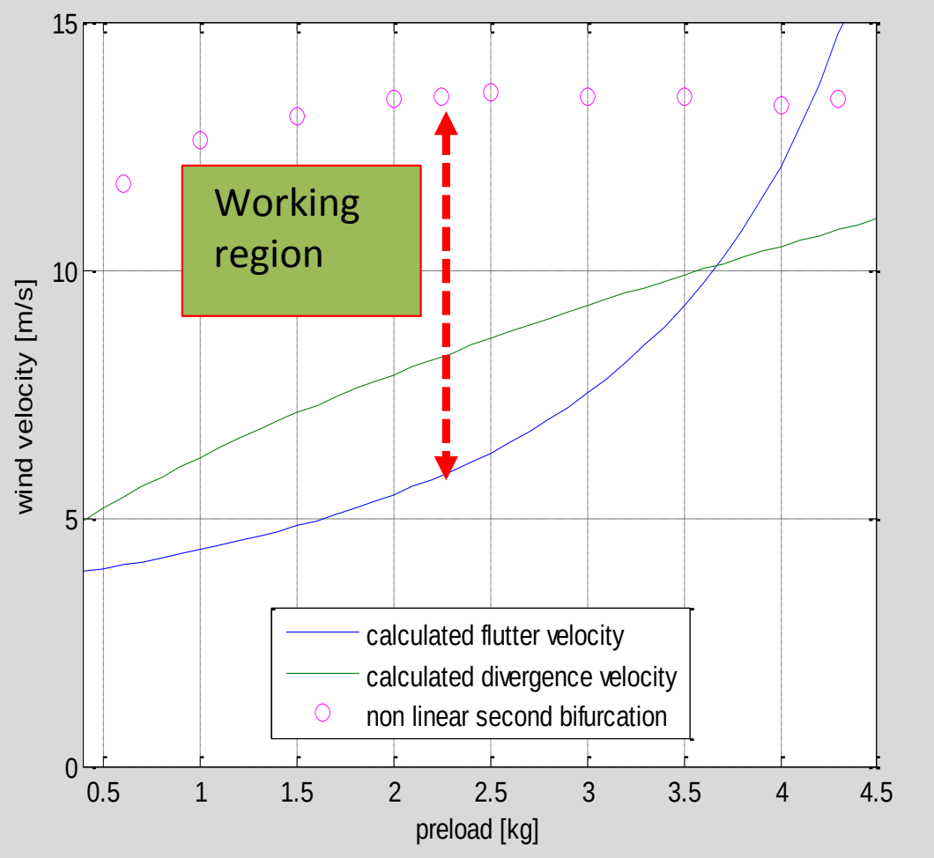
$$\overline{W} = \frac{1}{T}\sum_{i=1}^{4}\int_t^{t+T}\begin{pmatrix} FX_i(X_1,X_2,X_3,X_4)\cdot\dot X_i + \\ +F\dot X_i(\dot X_1,\dot X_2,\dot X_3,\dot X_4)\cdot\dot X_i \end{pmatrix} d\tau$$

Top boundary for the power harvesting.

- Optimal preload is dependent on designed velocity region
- Higher preloads yield smaller working region.



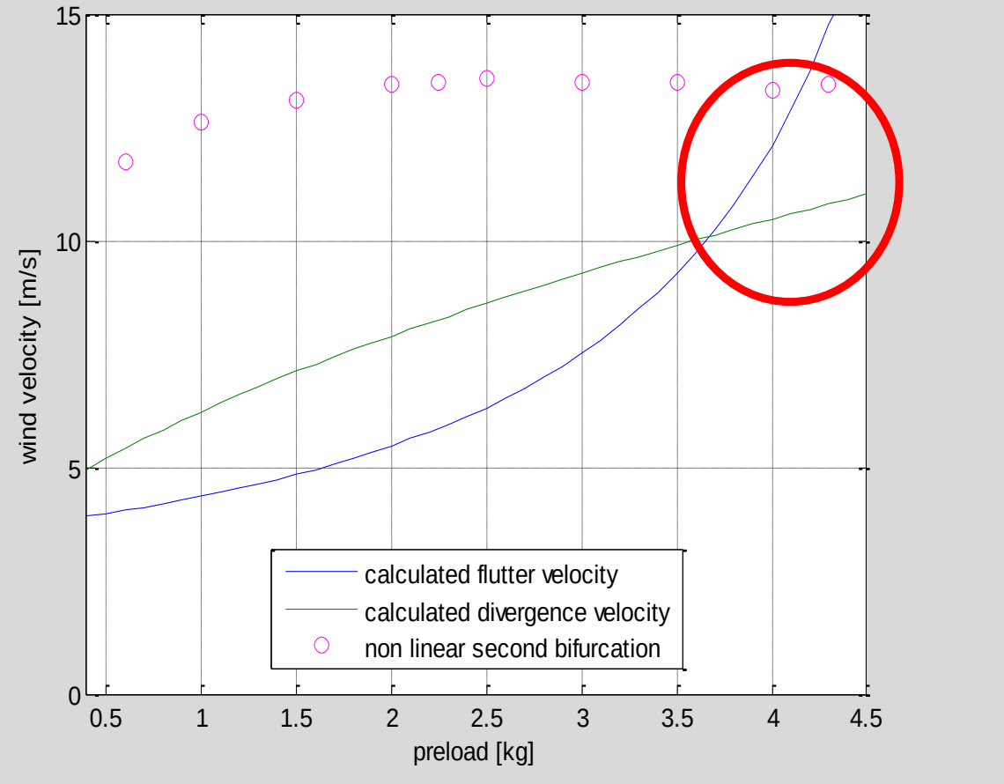
Power Vs airspeed for different preloads



Flutter, divergence and second stability loss Vs preload

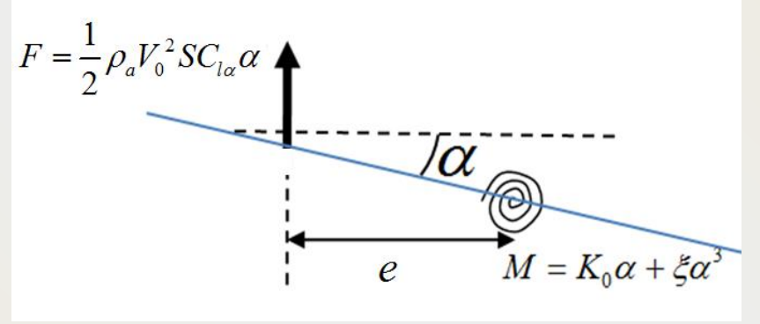
Divergence before flutter (?)

- At high preload values, linear analysis yields divergence before flutter
- Non linear analysis and experiment yield LCO at ~divergence velocity
- Assumed Mechanism:



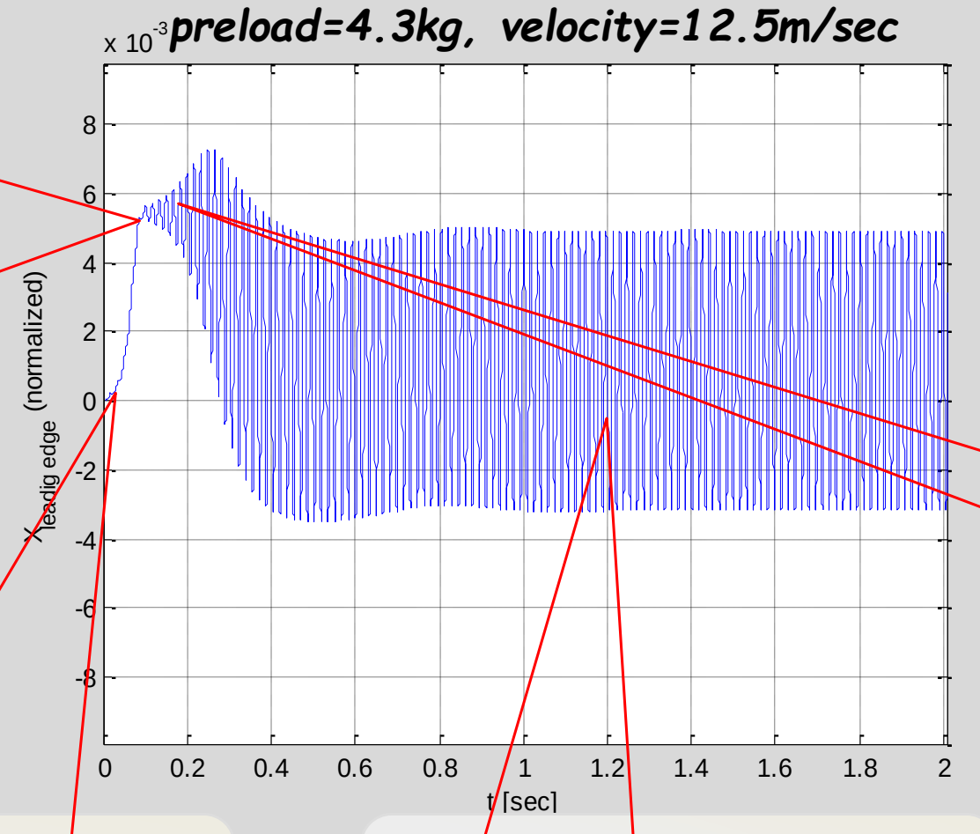
Flutter, divergence and second stability loss Vs preload

2. Static stabilization due to nonlinear stiffening



$$\alpha = \pm\sqrt{\left(\rho V_0^2 SeC_{L\alpha} - 2K_0\right)/2\xi}$$

Time history of the oscillation onset *preload=4.3kg, velocity=12.5m/sec*



1. Beginning of divergent movement

4. Flutter converges into LCO

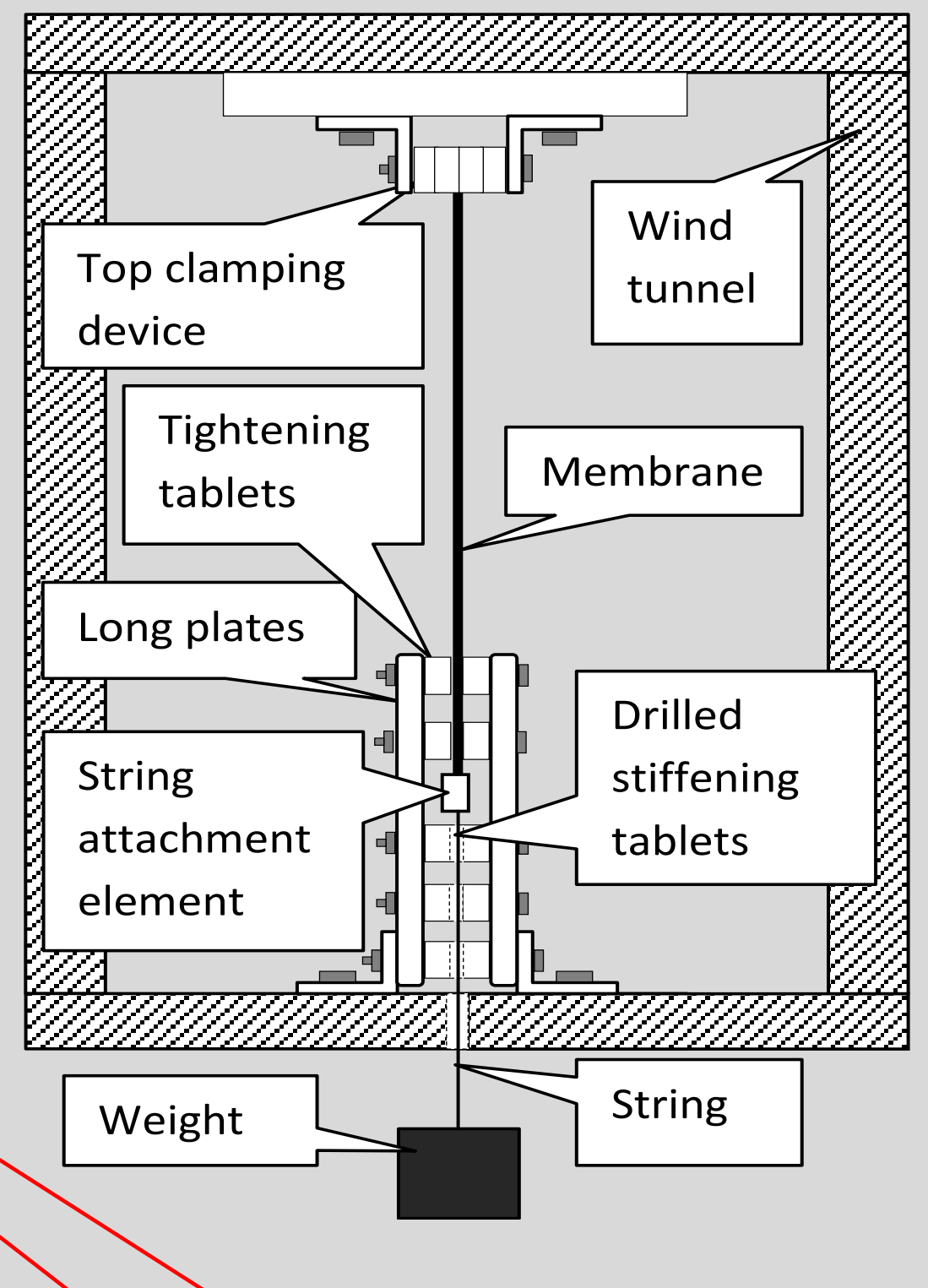
3. Dynamical system changes due to change in stiffness

$$K_{post\_divergenxe} = K_0 + \xi\alpha^2$$

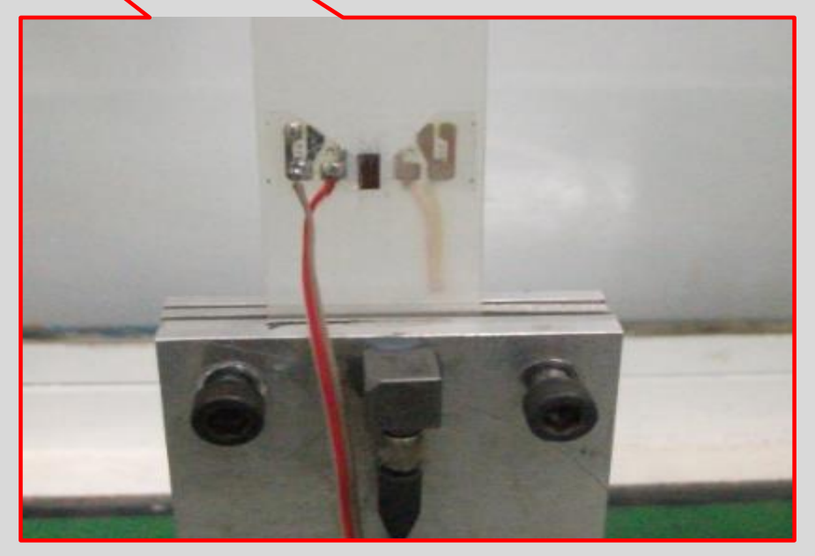
New system I s dynamically unstable leading to flutter

Experimental Study

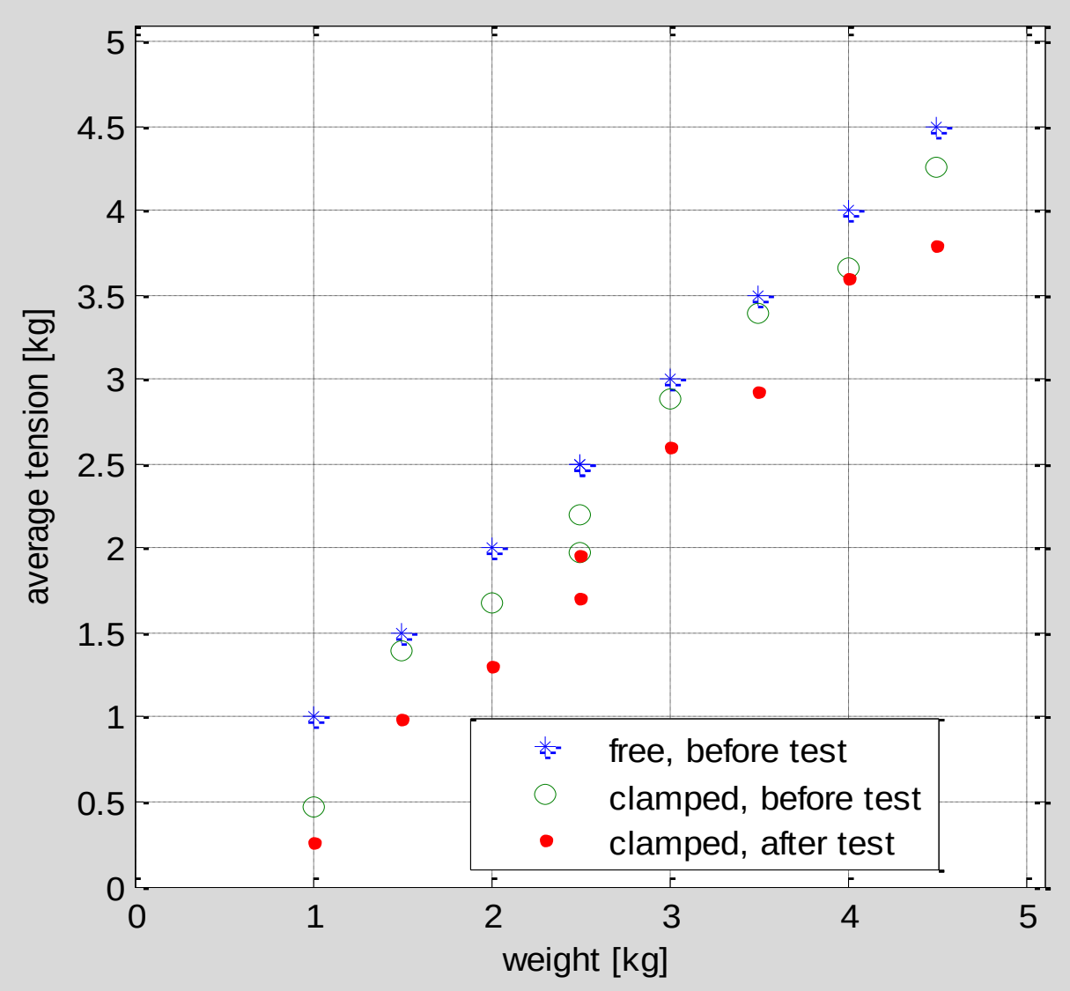
Test Setup



- 0.25/25/600 mm strip
- Weights: 0-5 kg
- Strain gauges ~50 mm from clamping device
- Accelerometer on clamping device
- Acquisition frequency 5 kHz
- Mode shape observed with a stroboscope



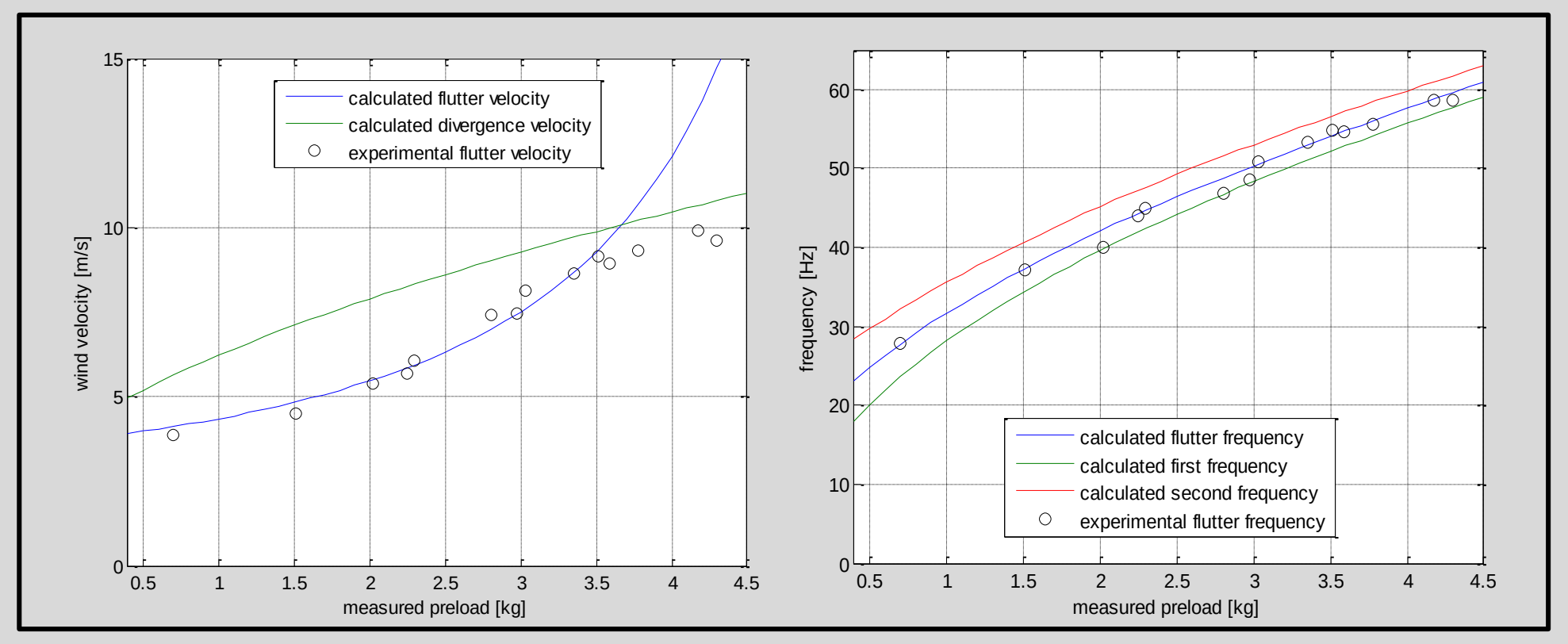
Preload measurement



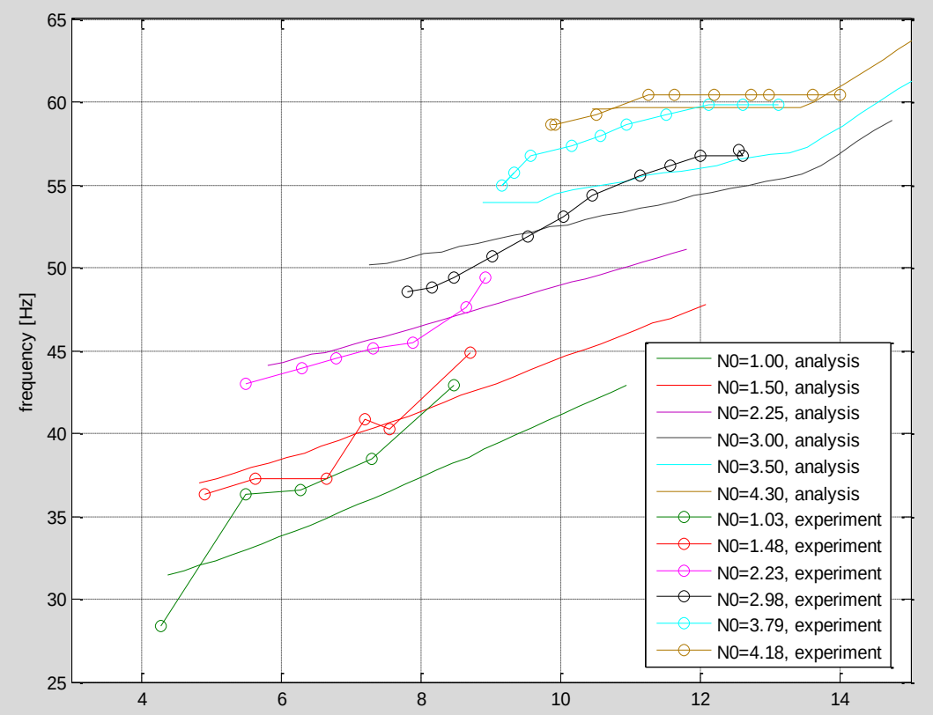
Measured preload at different stages of experiment Vs. hanged weight

- Preload is reduced during clamping
- Preload is reduced during Oscillation
- Results are based on the preload after clamping
- Results are more accurate at the beginning of the experiment

LCO onset and frequencies



Flutter velocities (left) and frequencies (right) as a function of preload – Comparison of experimental and numerical results



Post flutter frequencies: analysis Vs test

Strain amplitude comparison

Additional tension strain

$$\varepsilon_T = \frac{\Delta N}{Eh} = \frac{1}{2a}\int_0^a\left(\frac{\partial w(x,y)}{\partial x}\right)^2 dx = \frac{X_i^2}{2a}\int_0^a\left(\sum_{i=1}^{4}\left(\frac{\partial W_i}{\partial x} + y\frac{\partial\theta_i}{\partial x}\right)\right)^2 dx; \qquad f = 2f_{flutter}$$

Static pretension strain

$$\varepsilon_0 = N_0/AE; \qquad f = 0$$

$$\varepsilon = \varepsilon_0 + \varepsilon_b + \varepsilon_T$$

Bending strain

$$\varepsilon_b = \frac{\sigma}{E} = \frac{Mh}{2EI} = \frac{\partial^2 w}{\partial x^2}\frac{h}{2}; \qquad \frac{\partial^2 w}{\partial x^2}(x,t) \approx \sum_{i=1}^{4} X_i\left(\frac{\partial^2 W_i}{\partial x^2} + y\frac{\partial^2\theta_i}{\partial x^2}\right); \qquad f = f_{flutter}$$

Time history comparison of strains, *pretension=2.25 kg, airspeed=8.1 m/sec*