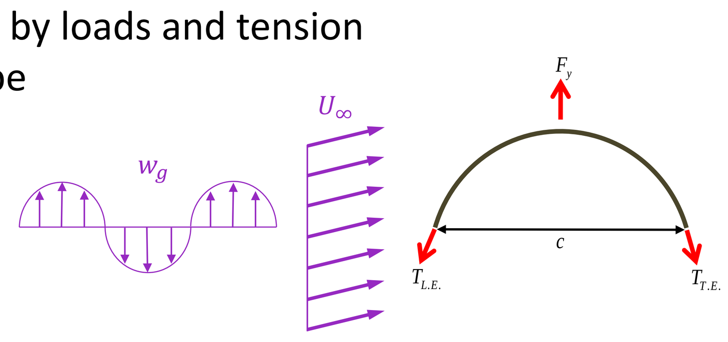
# Membrane Wing Gust Response

PhD Research Study by Sonya Tiomkin
Under the Guidance of Prof. Daniella Raveh

## Introduction

### Problem description

- Equilibrium of forces between the aerodynamic load, the membrane inertia and tension, where
  - Shape is determined by loads and tension
  - Loads vary with shape




### Main assumptions

- 2D
- Linearly elastic membrane
- Potential flow/Low Reynolds flow

### Objectives

- Establish knowledge of membrane wing dynamic response to gusts in terms of
  - Shape adaptation
  - Aerodynamic forces

## Mathematical Model

### Membrane model

Membrane dynamic equation

$$\rho h \frac{\partial^2 y}{\partial t^2} = T \frac{\partial^2 y}{\partial x^2}\left[1+\left(\frac{\partial y}{\partial x}\right)^2\right]^{-\frac{3}{2}} + \Delta p$$

Constitutive relation

$$T = T_0 + Eh\left(\frac{l-l_0}{l_0}\right)$$

Boundary conditions

$$y(x=0,t) = y(x=c,t) = 0$$

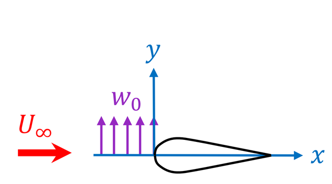
Initial conditions

$$y(x,t=0) = y_0(x)$$

$$\frac{\partial y}{\partial t}(x,t=0) = 0$$

### Gust model

Sharp-edge gust




$$w_g = \begin{cases} w_0 & \bar{x} < s \\ 0 & \bar{x} > s \end{cases} \quad \left(\bar{x} = \frac{x}{b}, s = \frac{U_\infty t}{b}, b = \frac{c}{2}\right)$$

Lift build-up in response to sharp-edge gust

$$L = 2\pi\rho_\infty U_\infty b w_0 \psi(s)$$

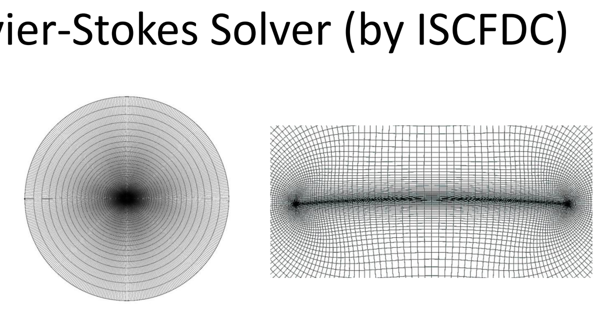
Kussner function approximation

$$\psi(s) \cong 1 - 0.5e^{-0.13s} - 0.5e^{-s}$$

## Methods

### Numerical solution

- Iterative procedure
- EZNSS - Elastic, Zonal, Navier-Stokes Solver (by ISCFDC)
- O-type grid (401x86)
- Laminar flow (Re=2500)



### Analytical solution

- Assumptions: small AoAs, small camber, potential flow
- Membrane slope represented by a Fourier series expansion with time dependent coefficients
- Unsteady thin airfoil theory used
- Added mass term omitted at first stage
- Solved for elastic/constant tension cases

## Contact Information

- sonya@tx.technion.ac.il
- Phone: 04-8293819

## Results

### Membrane dynamic solution

#### *Analytical results*

- 3rd mode oscillations obtained in unstable conditions
- Membrane inertia does not affect steady state solution in stable cases

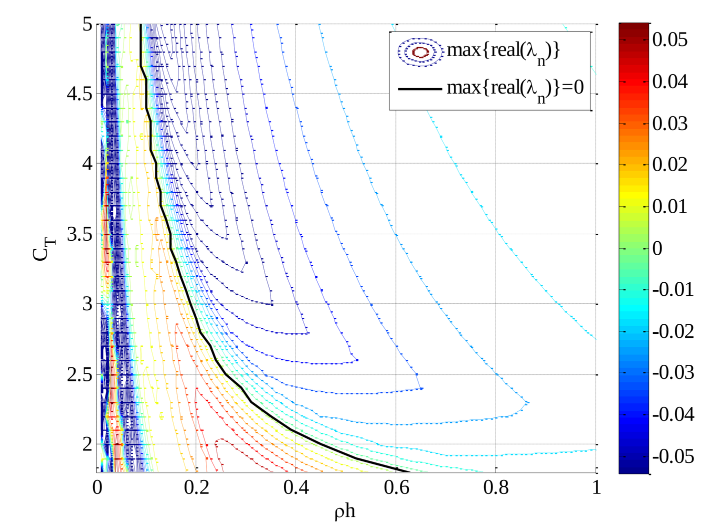



**Fig. 1** - Stability analysis of the analytical solution

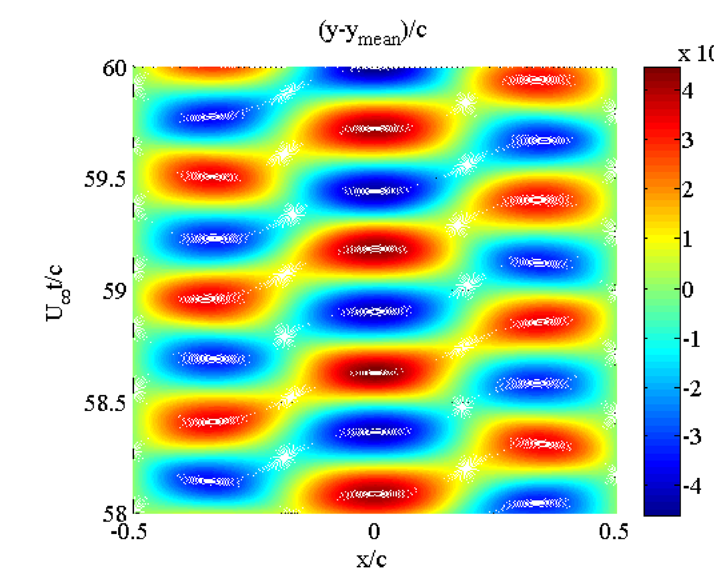
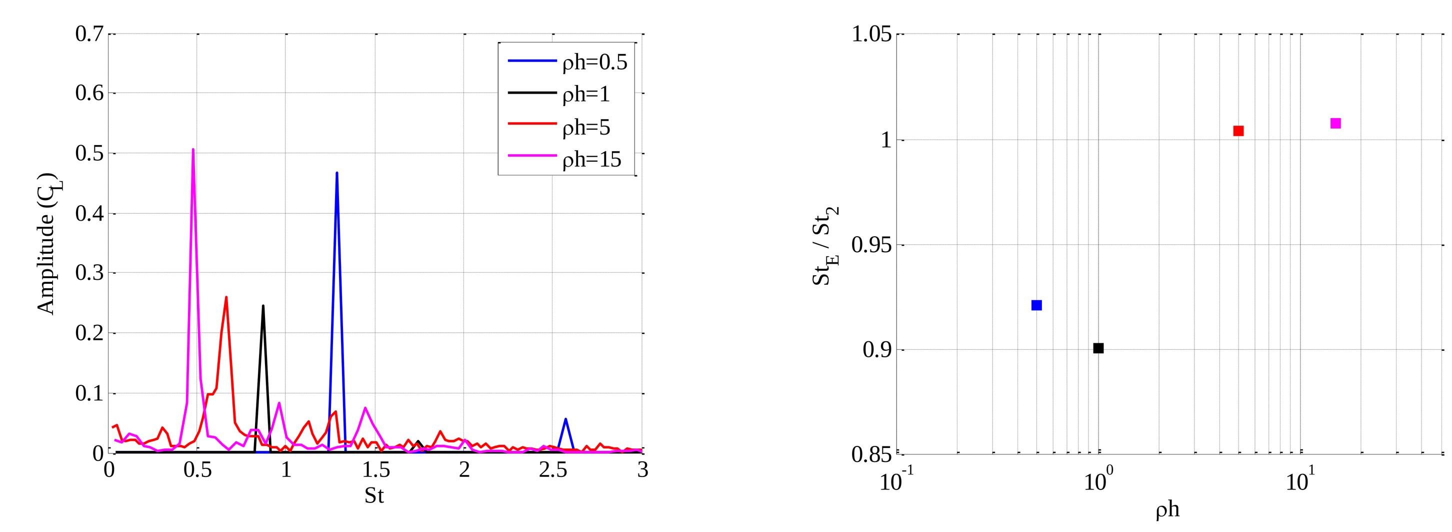



**Fig. 2** – Membrane shape perturbation from the mean profile, obtained with the elastic analytical solution for $\rho h = 0.5, C_{T_0} = 1, Eh = 50, \alpha = 2°$

#### *Numerical results*

- Strouhal number: $St = \frac{fc}{U_\infty}$
- Natural frequency of a string:

$$St_n = \frac{n}{2}\sqrt{\frac{C_T}{2\rho h}}$$




(a) Frequency response (b) Dominant frequency response

**Fig. 3** – Lift coefficient frequency response obtained with $C_{T_0} = 1.5, Eh = 50, \alpha = 4°$ and $Re = 2500$ for various mass values

### Rigid airfoil gust response in low Reynolds

- Lift drop is obtained when the gust front reaches the T.E. (at time $s = 2$) due to a momentary failure to meet the Kutta condition
- A thinner airfoil presents a response more similar to the Kussner function
- The airfoil camber does not affect the lift response

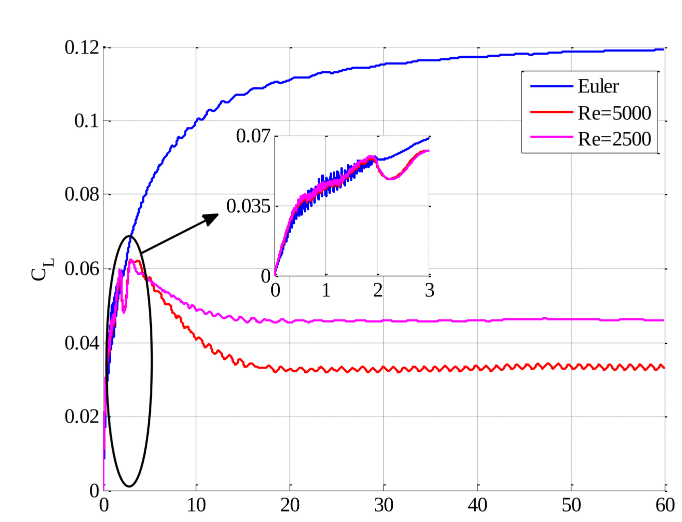



**Fig. 4** – Low Re effect on the lift response of a NACA0012 airfoil to a sharp-edge gust of 1°

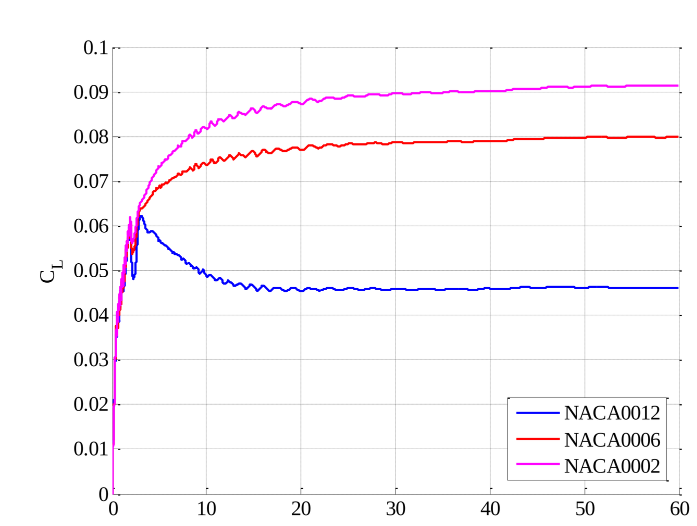



**Fig. 5** – Lift response of 3 symmetric NACA airfoils of different thickness to a sharp-edge gust of 1° at $Re = 2500$

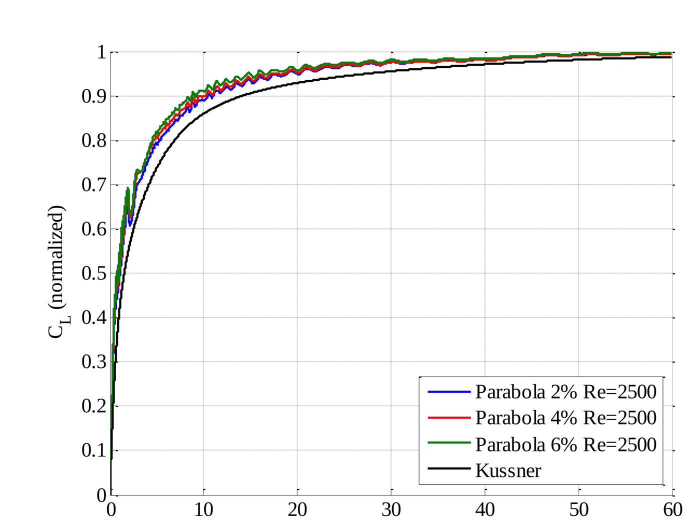
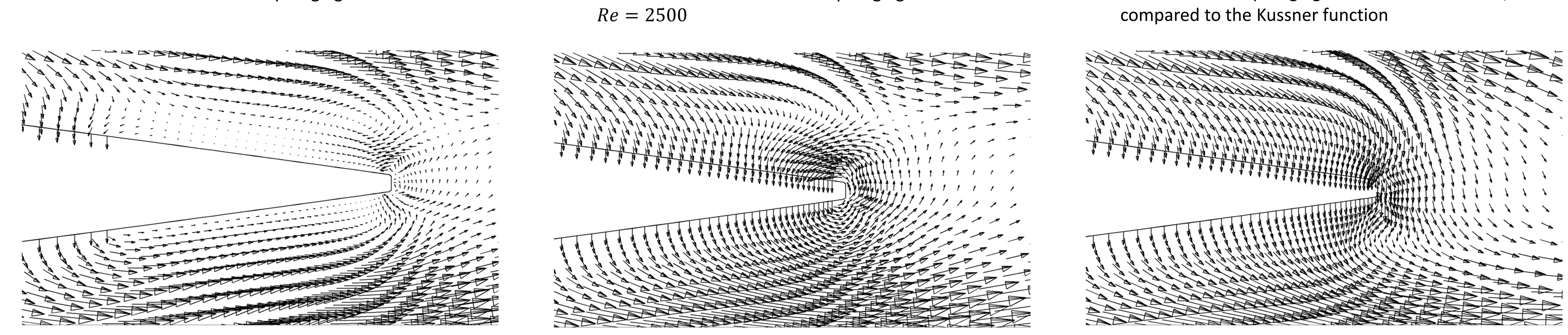



**Fig. 6** – Lift response of parabolic airfoils of different camber to a sharp-edge gust of 1° at $Re = 2500$, compared to the Kussner function



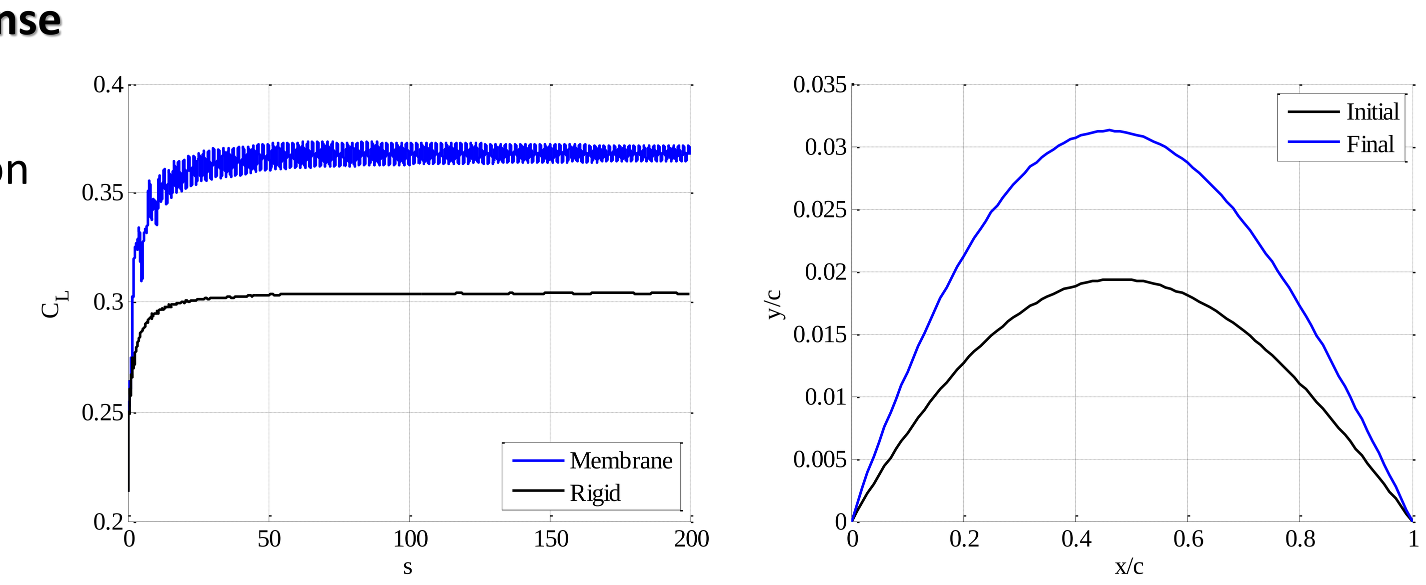
(a) $s = 1.92$ (b) $s = 2$ (c) $s = 2.24$

**Fig. 7** – Velocity vector field during gust penetration for a NACA0012 airfoil at $Re = 2500$ (T.E. area)

### Membrane wing sharp-edge gust response

- Membrane presents **no** lift drop at time $s = 2$ due to deformation of the T.E. area

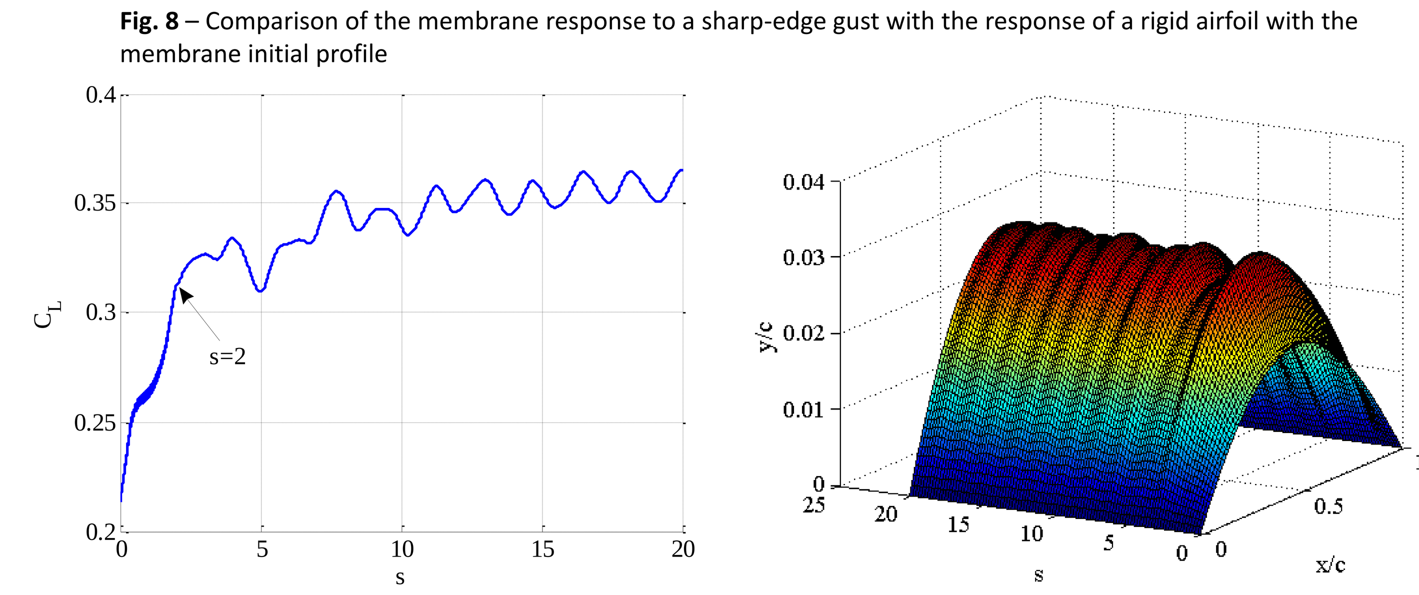
Scan to view a short movie of T.E. area velocity vectors




(a) Lift response (b) Membrane profile

**Fig. 8** – Comparison of the membrane response to a sharp-edge gust with the response of a rigid airfoil with the membrane initial profile

- Membrane parameters: $C_{T_0} = 1.5, Eh = 50, \rho h = 0.5$
- Flow parameters: $\alpha_0 = 1°, \alpha_g = 1°, Re = 2500$




(a) Lift response (b) Membrane shape response

**Fig. 9** – Membrane response to a sharp-edge at initial transient